PHYS-434 – Physics of photonic semiconductor devices, Raphaël Butté

Solution 10 – ABC model

Exercise I: Internal quantum efficiency

1. S1 has an InGaN underlayer before the QW: it has a **higher peak IQE** (reason 1) and this **peak IQE occurs at a lower current density** (reason 2). Both effects would be caused by a decrease in the A term in the ABC model (representing Shockley-Read-Hall recombination).

2. From the graph, we find:

$$\text{S1: } \eta_{\max} = 40.1\%,\ n_m(n\ @\ \eta_{\max}) = 2.9 \times 10^{12}\ \text{cm}^{-2}$$
$$\text{S2: } \eta_{\max} = 18.3\%,\ n_m = 6.0 \times 10^{12}\ \text{cm}^{-2}$$

A unit check is necessary at this point: we have n_m in units of cm^{-2}, but in the question C has been given in units of cm^6s^{-1}. Given that the relevant equation is:

$$\frac{n}{\tau} = An + Bn^2 + Cn^3 \Rightarrow \frac{1}{\tau} = A + Bn + Cn^2$$

If C is in cm^6s^{-1}, then n in this expression should be in cm^{-3} $\Rightarrow$ The C values in the question are 3D values, and we need n in cm^{-3}:

$$n_{3\text{D},m} = \frac{n_{2\text{D},m}}{t_{\text{QW}}}$$

$$\text{S1: } n_m = \frac{2.9 \times 10^{12}}{2.7 \times 10^{-7}} = 1.07 \times 10^{19}\ \text{cm}^{-3}$$
$$\text{S2: } n_m = \frac{6.0 \times 10^{12}}{2.7 \times 10^{-7}} = 2.22 \times 10^{19}\ \text{cm}^{-3}$$

By definition,

$$\eta = \frac{\text{generated photons}}{\text{injected electrons}} = \frac{\text{rate of radiative recombination}}{\text{total recombination rate}}$$

From the ABC model, radiative rate $= Bn$, total rate $= A + Bn + Cn^2$,

$$\Rightarrow \eta = \frac{Bn^2}{An + Bn^2 + Cn^3} = \frac{Bn}{A + Bn + Cn^2}$$

Differentiating:

$$\frac{\mathrm{d}\eta}{\mathrm{d}n} = \frac{B(A + Bn + Cn^2) - Bn(B + 2Cn)}{(A + Bn + Cn^2)^2}$$

$$\left.\frac{\mathrm{d}\eta}{\mathrm{d}n}\right|_{\eta=\eta_m} = 0 \Rightarrow 0 = AB + B^2 n_m + BCn_m^2 - B^2 n_m - 2BCn_m^2$$

$$\Rightarrow Cn_m^2 = A \Rightarrow n_m = \sqrt{\frac{A}{C}}$$

Hence,

$$\eta_{\max} = \frac{Bn_m}{A + Bn_m + Cn_m^2} = \frac{B\sqrt{\frac{A}{C}}}{A + B\sqrt{\frac{A}{C}} + C(\frac{A}{C})} = \frac{B\sqrt{\frac{A}{C}}}{2A + B\sqrt{\frac{A}{C}}}$$

$$\Rightarrow A = Cn_m^2 \text{ and } B = \frac{2A\eta_{\max}}{\sqrt{\frac{A}{C}}(1 - \eta_{\max})}$$

$$\text{S1: } A = (7 \times 10^{-32})(1.07 \times 10^{19})^2 = 8.0 \times 10^6 \text{ s}^{-1}$$

$$B = \frac{2(8.0 \times 10^6)(0.401)}{\sqrt{\frac{8\times10^6}{7\times10^{-32}}}(1 - 0.401)} = 1.0 \times 10^{-12} \text{ cm}^3\text{s}^{-1}$$

$$\text{S2: } A = (1 \times 10^{-31})(2.22 \times 10^{19})^2 = 4.9 \times 10^7 \text{ s}^{-1}$$

$$B = \frac{2(4.9 \times 10^7)(0.183)}{\sqrt{\frac{4.9\times10^7}{1\times10^{-31}}}(1 - 0.183)} = 9.9 \times 10^{-13} \text{ cm}^3\text{s}^{-1}$$

So, as we could expect the sample with the underlayer has a much lower A coefficient, since the underlayer has trapped point defects, avoiding their incorporation into the QW and reducing SRH recombination. Meanwhile, B is similar for both samples since their only difference is the underlayer, which is not expected to impact radiative processes.

3. By definition,

$$\tau_r = \frac{1}{Bn}, \tau_{nr} = \frac{1}{A + Cn^2}$$

Therefore, at maximum IQE:

$$\tau_r = \frac{1}{Bn_m} = \frac{1}{B}\sqrt{\frac{C}{A}}, \tau_{nr} = \frac{1}{A + Cn_m^2} = \frac{1}{A + C(\frac{A}{C})} = \frac{1}{2A}$$

$$\text{S1: } \tau_r = \frac{1}{1.0 \times 10^{-12}}\sqrt{\frac{7 \times 10^{-32}}{8.0 \times 10^6}} = 93.5 \text{ ns}$$

$$\tau_{nr} = \frac{1}{2(8.0 \times 10^6)} = 62.5 \text{ ns}$$

$$\text{S2: } \tau_r = \frac{1}{9.9 \times 10^{-13}}\sqrt{\frac{1 \times 10^{-31}}{4.9 \times 10^7}} = 45.6 \text{ ns}$$

$$\tau_{nr} = \frac{1}{2(4.9 \times 10^7)} = 10.2 \text{ ns}$$

The lowest lifetime represents the process that will dominate recombination dynamics – for both samples nonradiative processes dominate, which we could expect from as $\eta_{\max} < 50\%$ for both samples. We can see both lifetimes for S2 are shorter than for S1, which reflects the higher carrier densities for S2 at $\eta_{\max}$. A good check on our answer is to use them to calculate IQE and confirm what we expect:

$$\eta = \frac{\text{rate of radiative recombination}}{\text{total recombination rate}} = \frac{\frac{n}{\tau_r}}{\frac{n}{\tau_{nr}}} = \frac{\frac{1}{\tau_r}}{\frac{1}{\tau_r} + \frac{1}{\tau_{nr}}} = \frac{\tau_{nr}}{\tau_r + \tau_{nr}}$$

$$\text{S1: } \eta_{\max} = \frac{62.5}{62.5 + 93.5} = 40.1\% \checkmark$$

$$\text{S2: } \eta_{\max} = \frac{10.2}{10.2 + 45.6} = 18.3\% \checkmark$$

4.
$$\frac{J}{qt_{\mathrm{QW}}} = \frac{n}{\tau_{\mathrm{tot}}} = An + Bn^2 + Cn^3$$
$$\Rightarrow J|_{\eta=\eta_{\max}} = qt_{\mathrm{QW}}(An_m + Bn_m^2 + Cn_m^3)$$

S1: $J_m = (1.602 \times 10^{-19})(2.7 \times 10^{-7})(1.07 \times 10^{19})$
$\times [(8 \times 10^{6}) + (1 \times 10^{-12})(1.07 \times 10^{19}) + (7 \times 10^{-32})(1.07 \times 10^{19})^2] = 12.4\ \mathrm{Acm}^{-2}$

S2: $J_m = (1.602 \times 10^{-19})(2.7 \times 10^{-7})(2.22 \times 10^{19})$
$\times [(4.9 \times 10^{7}) + (9.9 \times 10^{-13})(2.22 \times 10^{19}) + (1 \times 10^{-31})(2.22 \times 10^{19})^2] = 116.9\ \mathrm{Acm}^{-2}$

So, the underlayer sample is nearly comparable to state-of-the-art LEDs, while S2 is not – as could be expected.

5. There are two main limitations of the ABC model:

- We are assuming that the A, B and C coefficients do not themselves depend on carrier density – this has been shown to not always be the case. For instance, at very high carrier densities SRH recombination can often become saturated, which leads to a reduction in the A term.
- The basics of the ABC model clearly depends on a free particle picture, i.e., the idea that SRH will depend on n as it is a single particle process, radiative recombination will depend on n^2 as it is a two-particle process, and so on. Yet, we know that carriers in semiconductors often form excitons, quasiparticles formed from two free particles, and so this "free particle picture" is not fully grounded in reality.

Further information:

Note that in addition to higher A, sample S2 also has higher C compared to S1. This is actually thought to be a linked effect, with current thinking being that point defects actually increase Auger recombination (potentially due to loosening of momentum conservation). This topic is currently under further research.

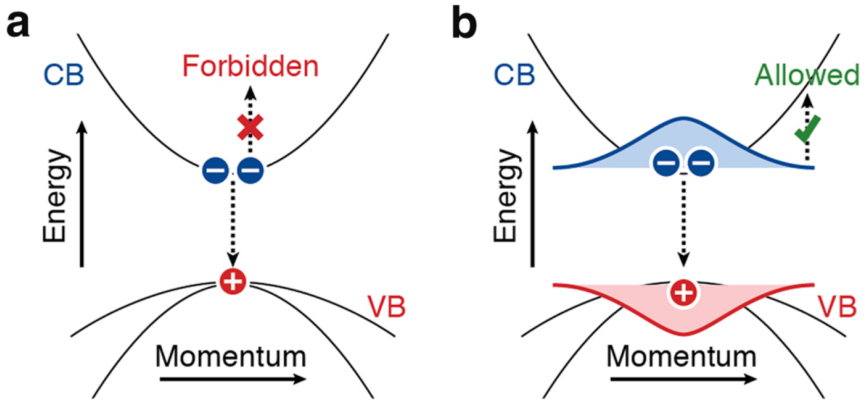

Figure 1: Momentum conservation in Auger processes: **a)** In a bulk semiconductor, recombination of an electron-hole pair at the band edge doesn't change translational momentum, thus excess energy cannot be absorbed by the electron. **b)** In the presence of point defects, carriers can be trapped (as in quantum dots) where momentum selection rule is less strict. This allows the Auger process to occur much faster. (Figure: F. T. Rabouw *et al. Top Curr Chem (Z)*, **58**, (2016))

Underlayer, see:

- Haller, C. *et al.* Burying non-radiative defects in InGaN underlayer to increase InGaN/GaN quantum well efficiency. *Appl. Phys. Lett.* **111**, 262101 (2017).
- Haller, C. *et al.* GaN surface as the source of non-radiative defects in InGaN/GaN quantum wells. *Appl. Phys. Lett.* **113**, 111106 (2018).
- Haller, C. *et al.* InAlN underlayer for near ultraviolet InGaN based light emitting diodes. *Appl. Phys. Express* **12**, 034002 (2019).

Exercise II: Time-resolved photoluminescence

1. m-QW2 clearly has a shorter lifetime at earlier time, while after longer time its carrier lifetime is more similar to m-QW1. Since early times correspond to higher carrier densities, this suggest that m-QW2 experiences higher recombination rate at high carrier densities, but a similar recombination rate at low carrier densities.

   At this stage, we cannot say whether the increased recombination rate is due to more efficient SRH, radiative or Auger processes, though the fact that it occurs at higher carrier densities suggests radiative or Auger processes since these are dependent on n^2 or n^3.

Note on carrier lifetime extraction:

We always have:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{n}{\tau_{\mathrm{tot}}}$$

This 1st order differential equation does not always have an analytical solution since τ_{tot} can depend on n: indeed the ABC model shows this is expected. However, we can simplify the problem by assuming that τ_{tot} doesn't change too rapidly with n, and only thinking of the rate over a small time range where τ_{tot} is nearly constant:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{n}{\tau_{\mathrm{tot,\,0}}}$$

$$\Rightarrow n = n_0 \exp\left(-\frac{t-t_0}{\tau_{\mathrm{tot,\,0}}}\right), \text{ with } n_0 = n(t=t_0)$$

Now, photoluminescence intensity represents the radiative recombination rate, and here we can also make an assumption that over our small time range, τ_r is about constant, $\tau_{r,0}$:

$$I \propto \frac{n}{\tau_{r,0}} = \frac{n_0}{\tau_{r,0}} \exp\left(-\frac{t-t_0}{\tau_{\mathrm{tot,\,0}}}\right)$$

Hence over a small time range, the gradient of a trPL $log(I)$ vs t plot should equal $1/\tau_{\mathrm{tot,\,0}}$. So, fitting the trPL decay over a small region within which the decay can be approximated as monoexponential allows us to extract the carrier lifetime for a given carrier density (Fig. 2).

There are limitations to this method however, and a more appropriate method is to fit the curve fully using the ABC model, especially if Bn^2 can be calculated more physically as $\int_{E_{\mathrm{QW}}}^{\infty} R_{\mathrm{spont}}(E)\mathrm{d}E$, where $R_{\mathrm{spont}}(E)$ is the full equation for spontaneous radiative emission at a given energy. For a good example of this method, and a full explanation and rigorous treatment of the results used in this question, see:

Shahmohammadi, M. *et al.* Enhancement of Auger recombination induced by carrier localization in InGaN/GaN quantum wells. *Phys. Rev. B* **95**, 125314 (2017).

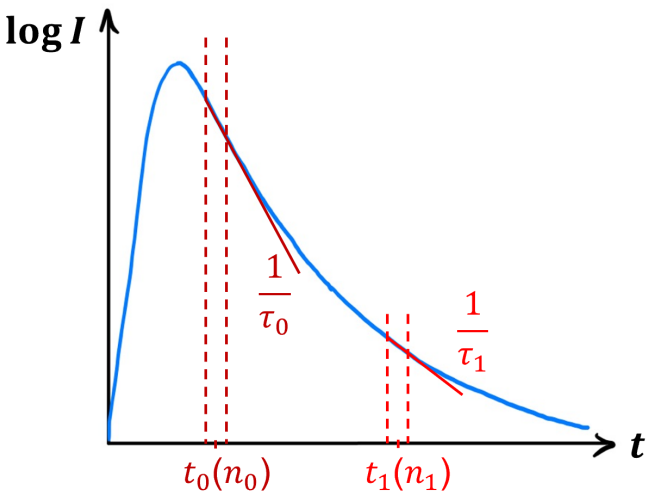

Figure 2: Extraction of the carrier lifetime for a given carrier density from trPL.

2. The relevant physical quantities are the defect density and the carrier diffusion length (which will depend on their lifetime and diffusion constant, $L \sim \sqrt{D\tau}$). Diffusion length is usually low at very low temperature, so in this limit it can often be assumed that the carriers are unable to diffuse to the defects/impurities at which SRH recombination occurs. However, this may not necessarily be true: A can still be significant at low temperature if the defect density or diffusion length are large enough. As such, this assumption should be made with caution.

3. Since we assume m-QW1 is purely radiative, the lifetime at early decay should be qual to the radiative lifetime:

$$m\text{-QW1: } \tau_{\text{tot, 0}} = \tau_{r,0} = 250 \text{ ps}$$

Then, at the same delay, m-QW2 has a similar carrier density and our assumption is it has similar radiative behaviors so:

$$m\text{-QW2: } \tau_{r,0} = 250 \text{ ps}$$

Then,

$$\text{IQE} = \frac{\frac{n_0}{\tau_{r,0}}}{\frac{n_0}{\tau_{\text{tot, 0}}}} = \frac{\tau_{\text{tot, 0}}}{\tau_{r,0}} = \frac{120 \text{ ps}}{250 \text{ ps}} = 48\%$$

Since we assume $A = 0$, and $B_{m-\text{QW1}} = B_{m-\text{QW2}}$, the difference in the behavior of the samples must come from Auger recombination. Auger goes as n^3, and so has a negligible impact at low n i.e. long time delays. So at long t, the IQE should tend to 100% for m-QW2.

4. $n_0 \simeq 7.5 \times 10^{12}$ cm^{-2}, $t_{\text{QW}} = 5$nm.

N.B. – This is a non-polar (m-plane) QW, so even though it is quite thick, there is no quantum confined Stark effect.

$$\frac{1}{\tau_{nr,0}} = \frac{1}{\tau_{\text{tot, 0}}} - \frac{1}{\tau_{r,0}} = \frac{1}{120} - \frac{1}{250} \Rightarrow \tau_{nr,0} = 230.8 \text{ ps}$$

$$\frac{1}{\tau_{nr,0}} = Cn_0^2 \Rightarrow C = \frac{1}{\tau_{nr,0} n_0^2}$$

$$\Rightarrow C = \frac{1}{(230.8 \times 10^{-12}) \left(\frac{7.5 \times 10^{12}}{5 \times 10^{-7}}\right)^2} = 1.9 \times 10^{-29} \text{ cm}^6\text{s}^{-1}$$

5. Increased laser pulse fluence corresponds to higher n_0 (more incident photons = more generated electrons). Since

$$\frac{1}{\tau_{\text{tot, 0}}} = \frac{1}{\tau_{r,0}} + \frac{1}{\tau_{nr,0}} = Bn_0 \text{ for } m\text{-QW1}$$
$$= Bn_0 + Cn_0^2 \text{ for } m\text{-QW2}$$

$\tau_{\text{tot, 0}}$ is expected to have a stronger dependence on n_0 for m-QW2 compared to m-QW1, and hence on laser fluence. This explains the results (Linked to Q3).

N.B. – Simplifying assumptions have been made for this question: in reality, B is not the same for both samples. The samples differ in term of alloy disorder. This can both increase B and C. For further information, see the citation earlier in this question